Method for improving the quality of images acquired by chirped pulse phase-shifting digital holography

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Abstract: We have been studying the methods for acquiring the amplitude images and the phase images of ultrafast phenomena such as laser processing and discharge in femtosecond time resolution using an ultrashort optical pulse from a laser using chirped pulse phase-shifting digital holography. By using this method, it is possible to acquire the amplitude images and the phase images simultaneously at multiple times. However, to acquire them, it is necessary to acquire the images with different wavelength by changing the tilt angle of a band pass filter (BPF). When the BPF is tilted, the polarization state of the reference light is modified, and it causes the deterioration of the acquired images. In this study, we developed the method to correct the polarization state of the reference light for improving the quality of acquired images. We defined two parameters α and θ from the recorded intensity of the reference light, and the state of the polarized chirped reference light was written using the Jones vector with these two parameters. When the amplitude images and the phase images were reconstructed using these parameters, the quality of the acquired images was improved.
Key words: phase-shifting digital holography, chirped pulse, band pass filter

1. INTRODUCTION

We have been studying the methods for acquiring the amplitude images and the phase images of ultrafast phenomena such as laser processing and discharge in femtosecond time resolution using an ultrashort optical pulse from a laser using chirped pulse phase-shifting digital holography [1].

In phase-shifting digital holography, multiple holograms are recorded and the phase of reference light is changed at different recordings. The complex amplitude distribution of only the object light is calculated from the recorded holograms. By using this method, the spatial resolution can be better than off-axis digital holography [2].

A chirped pulse is a pulse whose frequency changes over time. When light passes through glass, the group velocity of the high-frequency components of the light wave becomes slower than the low-frequency components of the light wave due to the group velocity dispersion of the glass, resulting in a positive chirp where the frequency increases over time. By chirping the reference light and dividing the object light with different optical path length, holograms at multiple different timings can be recorded in a single shot. In this experiment, single object light is used.

In Fig.1, experimental setup for chirped pulse phase-shifting digital holography used in this study is shown.

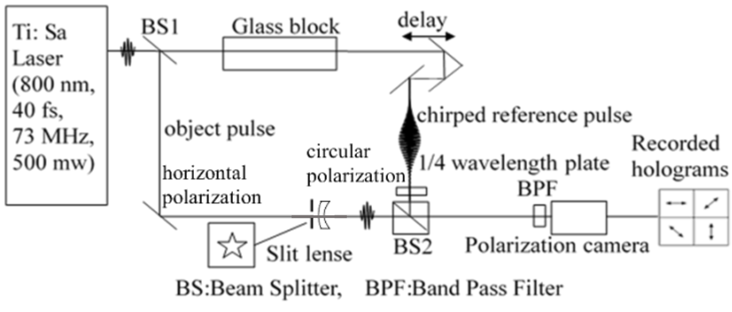

Fig.1 Experimental setup for chirped pulse phase-shifting digital holography.

The ultrashort pulse laser light emitted from a titanium-sapphire laser is divided into the object light and the reference light by a BS1. The object light passes through the star-shaped slit and the concave lense. The reference light is given a large chirp by passing through a glass block, and becomes circularly, polarized light by passing through a 1/4 wavelength plate. The object light and the reference light are recombined at a BS2, and these are recorded by a polarization camera that can record intensity images in four different linear polarization directions ($0^{\circ}, 90^{\circ}, 45^{\circ}$, and 135°). A BPF allows only light of a specific wavelength to pass through.

By dividing object light, it is considered that it possible to acquire the images at multiple different timings using a single-shot optical pulse, but in order to acquire them with different wavelength, it's necessary to acquire them by changing the tilt angle of the BPF. The BPF is used to select the wavelength of light and its transmission wavelength becomes shorter as the angle of incidence increases [3]. When the BPF is tilted, the polarization state of the reference light is modified, and it causes the deterioration of the acquired images.

In this study, we developed the method to correct the polarization state of the reference light for improving the quality of acquired images.

2. METHOD

2.1 Calculation of the corrected images

The circularly polarized chirped reference light can be written by Jones vector as shown below [4]

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}. \tag{1}$$

We introduce parameters α and θ in Eq. (1) as follows, to account for the effects of the tilt of a BPF,

$$\frac{1}{\sqrt{1+\alpha^2}}\begin{bmatrix}\alpha\\ e^{i\theta}\end{bmatrix}. \tag{2}$$

By assuming the polarization state of reference light as Eq. (2), the intensities of the reference light acquired by a polarization camera can be calculated as follows, where $R0$, $R90$, $R45$, and $R135$ are the intensities of reference light with polarization of 0°, 90°, 45°, and 135°, respectively and R is the amplitude.

$$R0 = \frac{\alpha^2|R|^2}{1+\alpha^2}, \tag{3}$$

$$R90 = \frac{|R|^2}{1+\alpha^2}, \tag{4}$$

$$R45 = \frac{|R|^2}{2+2\alpha^2}(\alpha^2 + 2\alpha\cos\theta + 1), \tag{5}$$

$$R135 = \frac{|R|^2}{2+2\alpha^2}(\alpha^2 - 2\alpha\cos\theta + 1). \tag{6}$$

From these equations, α and θ can be estimated as follows

$$\alpha = \sqrt{\frac{R0}{R90}}, \tag{7}$$

$$\theta = \cos^{-1}\left(\frac{0.5\alpha(R45 - R135)}{R0}\right). \tag{8}$$

By using the corrected polarization state of reference light in Eq. (2) instead of Eq. (1), where α and θ are given by Eqs. (7) and (8), the improvement of the quality of the reconstructed image is expected.

When the reference light is polarized as shown in Eq. (2) and the object light is linearly horizontally polarized, we can write the polarization of the combined electric field using the Jones vector as follows. Here, O and R are the complex electric field of the object light and reference light respectively,

$$O\begin{bmatrix}1\\0\end{bmatrix}+\frac{R}{\sqrt{1+\alpha^2}}\begin{bmatrix}\alpha\\e^{i\theta}\end{bmatrix}=\begin{bmatrix}O+\dfrac{\alpha R}{\sqrt{1+\alpha^2}}\\ \dfrac{e^{i\theta}R}{\sqrt{1+\alpha^2}}\end{bmatrix}. \tag{9}$$

The intensities I_1, I_2, I_3, and I_4 recorded by a polarization camera can be written as follows,

$$I_1=|O|^2+\frac{\alpha O R^*}{\sqrt{1+\alpha^2}}+\frac{\alpha O^* R}{\sqrt{1+\alpha^2}}+\frac{\alpha^2|R|^2}{1+\alpha^2}, \tag{10}$$

$$I_2=\frac{|R|^2}{1+\alpha^2}, \tag{11}$$

$$I_3=\frac{|O|^2}{2}+\frac{OR^*}{2\sqrt{1+\alpha^2}}\left(\alpha+e^{-i\theta}\right)+\frac{O^*R}{2\sqrt{1+\alpha^2}}\left(\alpha+e^{i\theta}\right)+\frac{|R|^2}{2+2\alpha^2}\left(\alpha^2+\alpha e^{i\theta}+\alpha e^{-i\theta}+1\right), \tag{12}$$

$$I_4=\frac{|O|^2}{2}+\frac{OR^*}{2\sqrt{1+\alpha^2}}\left(\alpha-e^{-i\theta}\right)+\frac{O^*R}{2\sqrt{1+\alpha^2}}\left(\alpha-e^{i\theta}\right)+\frac{|R|^2}{2+2\alpha^2}\left(\alpha^2-\alpha e^{i\theta}-\alpha e^{-i\theta}+1\right), \tag{13}$$

where I_1, I_2, I_3, and I_4 correspond to the polarization angles of 0°, 90°, 45°, and 135°, respectively.

From Eqs. (10), (11), (12), and (13), when the chirped reference light is circularly polarized $R=|R|$ and the object light is horizontally polarized $O=|O|e^{i\varphi}$, the images recorded at each pixel of the camera are written as follows,

$$I_1=|O|^2+\frac{\alpha^2|R|^2}{1+\alpha^2}+\frac{2\alpha\eta|O||R|}{\sqrt{1+\alpha^2}}\cos\varphi\,, \tag{14}$$

$$I_2=\frac{|R|^2}{1+\alpha^2}, \tag{15}$$

$$I_3=\frac{|O|^2}{2}+\frac{|R|^2}{2+2\alpha^2}(\alpha^2+2\alpha\cos\theta+1)+\frac{\eta|O||R|}{\sqrt{1+\alpha^2}}\big((\alpha+\cos\theta)\cos\varphi+\sin\theta\sin\varphi\big), \tag{16}$$

$$I_4=\frac{|O|^2}{2}+\frac{|R|^2}{2+2\alpha^2}(\alpha^2-2\alpha\cos\theta+1)+\frac{\eta|O||R|}{\sqrt{1+\alpha^2}}\big((\alpha-\cos\theta)\cos\varphi-\sin\theta\sin\varphi\big), \tag{17}$$

where the factor η that determines the intensity of the interference terms are included [1].

From Eqs. (14), (15), (16), and (17), we define A and B as follows,

$$\mathrm{A}=\frac{\sqrt{1+\alpha^2}}{2\alpha\eta}(I_3+I_4-|R|^2)=\frac{\sqrt{1+\alpha^2}}{2\alpha\eta}|O|^2+|O||R|\cos\varphi\,, \tag{18}$$

$$\begin{aligned}\mathrm{B}&=\frac{\sqrt{1+\alpha^2}}{2\alpha\eta\sin\theta}\left((\alpha-\cos\theta)I_3-(\alpha+\cos\theta)I_4+\frac{1-\alpha^2}{1+\alpha^2}|R|^2\cos\theta\right)\\&=-\frac{\sqrt{1+\alpha^2}\cos\theta}{2\alpha\eta\sin\theta}|O|^2+|O||R|\sin\varphi\,.\end{aligned} \tag{19}$$

From these equations, we can obtain the quadratic equation of $X=|O|^2$ after calculating $\mathrm{A}^2+\mathrm{B}^2$ as follows,

$$\frac{1+\alpha^2}{4\alpha^2\eta^2\sin^2\theta}X^2+\left(\frac{\sqrt{1+\alpha^2}}{\alpha\eta\sin\theta}(B\cos\theta-A\sin\theta)-|R|^2\right)X+A^2+B^2=0. \tag{20}$$

From Eq. (20), X can be obtained as follows using quadratic formula,

$$X = -\frac{2\alpha\eta\sin\theta}{\sqrt{1+\alpha^2}}(B\cos\theta - A\sin\theta) + \frac{2\alpha^2\eta^2\sin^2\theta}{1+\alpha^2}|R|^2$$
$$-\frac{2\alpha^2\eta^2\sin^2\theta}{1+\alpha^2}\sqrt{\left(\frac{\sqrt{1+\alpha^2}}{\alpha\eta\sin\theta}(B\cos\theta - A\sin\theta) - |R|^2\right)^2 - \frac{1+\alpha^2}{\alpha^2\eta^2\sin^2\theta}(A^2+B^2)}. \quad (21)$$

From Eq. (21), the object light O can be obtained as follows,

$$O = \frac{1}{|R|}\left(A + iB - \frac{\sqrt{1+\alpha^2}}{2\alpha\eta}X + i\frac{\sqrt{1+\alpha^2}\cos\theta}{2\alpha\eta\sin\theta}X\right). \quad (22)$$

2.2 The experimental setup of this study

In this study, we use the experimental setup in which a BPF in Fig.1 is tilted by 20°. In fig.2, the detailed experimental setup showing a BS2 and the BPF is shown.

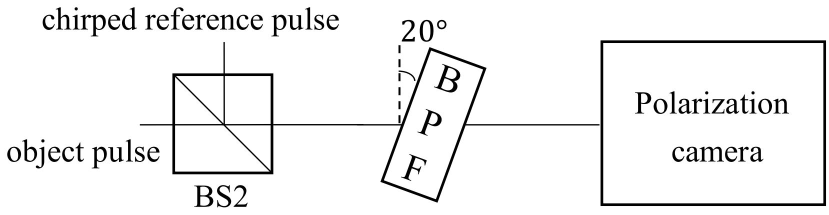

Fig.2 The detailed experimental setup of this study showing a BS2 and a BPF.

The hologram was recorded using the setup shown in Figs. 1 and 2 using the method mentioned above. When reconstructing the images, the method using the reference light shown in Eq. (2) is compared with the method using the reference light shown in Eq. (1) to study if the image with improved quality can be obtained by using the method of image reconstruction using Eqs. (2) - (22).

2.3 RMS Error

The accuracy of the image reconstruction is evaluated by the Root Mean Squared Error (RMSE), which is the square root of the mean of the squared differences between the intensity images of the object light before and after reconstruction. This value becomes lower as the quality of the reconstructed image improves.

We set the intensity of the object light before reconstruction at each pixel to be a_{ij}, and that after reconstruction to be b_{ij}, and RMSE to be I_e in the following. Since the images are composed of 2048×2048 pixels, the sum of the squared differences between the intensity of the object light before reconstruction and that after reconstruction is written as follows,

$$\sum_{i=1}^{2048}\sum_{j=1}^{2048}\left(a_{ij} - \mu b_{ij}\right)^2 = s, \quad (23)$$

Where the scaling factor μ is written as follows,

$$\mu = \sum_{i=1}^{2048}\sum_{j=1}^{2048}\frac{a_{ij}b_{ij}}{{b_{ij}}^2}. \quad (24)$$

From these values, I_e can be obtained as follows,

$$I_e = \sqrt{\frac{s}{2048 \times 2048}}. \quad (25)$$

2.4 Degree of Linear Polarization

We can calculate the Degree of Linear Polarization (DoLP) of the reference pulse only directly using the intensities of reference light $R0', R90', R45', and\ R135'$ with polarization of 0°, 90°, 45°, and 135° as follow,

$$\mathrm{DoLP}_1 = \frac{\sqrt{(R0' - R90')^2 + (R45' - R135')^2}}{R0' + R90'}. \tag{26}$$

This value becomes 0 when the reference light is circularly polarized. By using the two parameters defined by Eqs. (7) and (8), DoLP can be calculated by these parameters as follows,

$$\mathrm{DoLP}_2 = \frac{\sqrt{(\alpha^2 - 1)^2 + 4\alpha^2(\cos\theta)^2}}{1 + \alpha^2}. \tag{27}$$

By calculating DoLP_1, we can find how the reference light is different from the circularly polarized light. Also, by comparing DoLP_1 and DoLP_2, the we can find how well DoLP can be estimated by two parameters α and θ.

3. EXPERIMENTAL RESULT

In Fig. 3 (a) and (b), the reconstructed amplitude image and the phase image without correction are shown. In Fig. 3 (c) and (d), the reconstructed amplitude image and the phase image with correction are shown similarly. These images are reconstructed at the position of the star-shaped slit in Fig. 1 using the Fresnel transformation [5]. The parameters calculated for the corrections in Eq. (2) obtained using Eqs. (7) and (8) are determined to be α =1.17 and θ =81.8°.

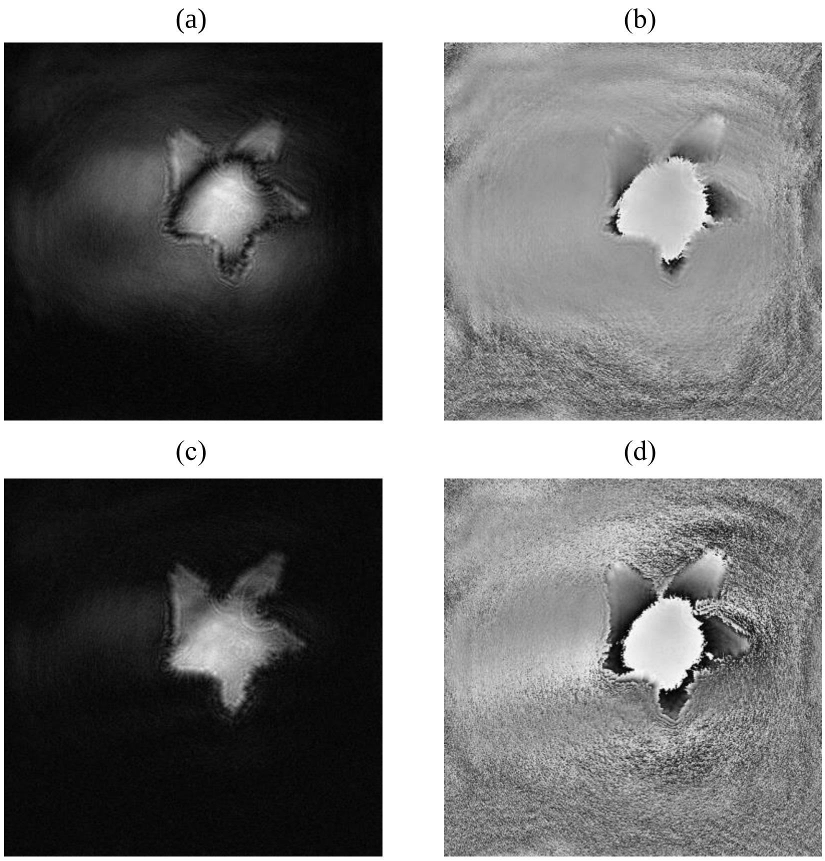

Fig.3 (a) the amplitude image without correction. (b) the phase image without correction. (c) the amplitude image with correction. (d) the phase image with correction. Four images size are 7.07 × 7.07 mm. Four images are greyscale, the white part has a phase of 2π and the black part has a phase of 0.

First, from the reconstructed image without using correction in Fig.3 (a), it is observed that noticeable interference fringes and background light appear in the amplitude image, although the star-shaped object is reconstructed. Also, from Fig.3 (b), it is observed that the object has a downward convex phase since the convex lense is used, However, the phase values of 0 and 2π can hardly be confirmed. On the other hand, from the reconstructed image using correction in In Fig.3 (c), it is observed that there are almost no interference fringes and background light, and the star-shaped object can be clearly reconstructed. Also, from Fig.3 (d), it is observed that the object has the downward convex phase whose phase ranges from 0 to 2π. From Fig.3 (a), (b), (c), and (d), it can be seen that the quality of the amplitude image and the phase image is improved visually.

Next, RMSE of the reconstructed images are evaluated. The RMSE of the images without correction is 0.0593 and that with correction is 0.0352. From these values, it can be confirmed that the reconstructed image is improved.

Finally, the values of DoLP of the reference light are evaluated. The $DoLP_1$ and $DoLP_2$ of the reference light are evaluated to be 0.219 and 0.210, respectively. From these values, it can be confirmed that the polarization state of reference pulse is different from circular and it is well represented by the Jones vector given in Eq. (2).

4. CONCLUSION

From the comparison of images without correction in Fig.3 (a) and (b) and those with correction in Fig. 3 (c) and (d), it can be seen that the quality of the reconstructed amplitude image and that phase image is improved. And it is confirmed by calculating the RMSE of these images. Therefore, the method we developed can be used for correcting the polarization state of the reference light for improving the quality of acquired images.

In this study, we used a single object light pulse, but we plan to reconstruct multiple object images with different timings using a single-shot pulse.

5. REFERENCES

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